ECN 119: Economics and Psychology
Risk

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Risk

What does it mean to choose the right thing when you don't even know what the consequences will be? The world is a risky place and so economics needs to understand what kinds of decisions people make before the dice are rolled.

In this section we will discuss the history and mechanics of the canonical model of Expected Utility Theory, see where experiments have revealed its weak points, and what we can do to shore it up or replace it entirely. We will learn how to measure someone's attitude towards risk and why it matters both in economics experiments and the economy.

Risk

In this section:

1. Expected Utility Theory
2. The Allais Paradox and the Ellsberg Paradox
3. Risk aversion and measuring riskiness
4. Prospect Theory
5. Loss aversion
6. Information aversion
7. Reference dependence
8. Subjective Expected Utility
9. Rank Dependent Utility
10. Maxmin Expected Utility

The St. Petersburg Paradox

You have the chance to play a game. You will pay some amount of money to play, and you'll win some money based on how many tosses of a coin will be needed before it first turns up heads. If the coin first turns up heads on the nth toss, you receive \$$2^n$. That is, \$2 if it comes up on the first throw, \$4 if on the second, \$8 if on the third, and so on.

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How much would you pay to play this game?

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What is the expected value of the prize?

The St. Petersburg Paradox

Expected value of your prize:

$$E(prize) = \frac{1}{2}2 + \frac{1}{4}4 + \frac{1}{8}8 + ... \quad (1)$$

$$= 1 + 1 + 1... \quad (2)$$

$$= \infty \quad (3)$$

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What's going on here?

Choice under uncertainty

- Goal: a model of the choice of an individual faced with actions whose consequences are uncertain.
- Why: to plug in to bigger models of the various real-world settings which involve uncertainty. Examples?
- Canonical model we'll look at first is **Expected Utility**.
- Today we will learn what Expected Utility theory is and challenge its robustness. We'll also look at the concept of risk aversion. Next time we'll look at evidence on individual choice behavior and look at alternative modeling approaches that attempt to account for such evidence.

Lotteries

- Decision-maker (DM) faces a choice among risky alternatives.
- $\mathcal{C}$: set of all possible outcomes. The number of outcomes in $\mathcal{C}$ is finite, and they are indexed $1, ..., n$.
- **Simple lottery**: $L = (p_1, ..., p_n)$, $p_n \geq 0$ for all n, $\sum_n p_n = 1$.
- p_n: probability of outcome n occurring.
- **Compound lottery**: allows outcomes of a lottery to be simple lotteries.
- **Reduced lottery**: for a given compound lottery, the simple lottery that generates the same ultimate distribution over outcomes.

Preferences over lotteries

- **Consequentialism**: only the reduced lottery over final outcomes matters to the DM
- $\mathcal{L}$: set of all simple lotteries over outcomes $\mathcal{C}$.
- Assume DM has a rational preference relation $\succsim$ on $\mathcal{L}$ (**complete** and **transitive**).
- Keep in mind flexible definition of objects in $\mathcal{C}$!

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- **Continuity** (suppressing technical definition): small changes in probabilities don't alter the nature of the DM's ordering between two lotteries.

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- **Independence**: $\succsim$ over $\mathcal{L}$ satisfies the independence axiom if for all L_1, L_2, L_3 and $p \in (0,1)$:

$L_1 \succsim L_2$ if and only if $pL_1 + (1-p)L_3 \succsim pL_2 + (1-p)L_3$

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Preference over mixture of each of two lotteries with some third lottery follows preferences over the two lotteries themselves - it's independent of what third lottery we choose.

The expected utility form

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$U : \mathcal{L} \to \mathbb{R}$ has an expected utility form if there is an assignment of numbers to the N outcomes such that for every simple lottery $L \in \mathcal{L}$ we have

$$U(L) = u_1 p_1 + ... + u_N p_N \tag{4}$$

U with expected utility form is a **von Neumann-Morgenstern expected utility function**.

- Utility of a lottery is the expected value of the utility of the outcomes
- Linear function of probabilities
- EU property is *cardinal* property: magnitudes of utilities mean something here

The Expected Utility Theorem

Suppose that rational preference relation $\succsim$ satisfies the continuity and independence axioms. Then $\succsim$ admits a utility representation of expected utility form, so that it is possible to assign a number u_n to each outcome $n = 1, \dots, N$ such that $L \succsim L'$ if and only if $\sum_{n=1}^{N} u_n p_n \geq \sum_{n=1}^{N} u_n p'_n$.

- Indifference curves that represent this graphically are straight, parallel lines
- Is EU theory a 'good' model of choice under uncertainty? Does it make concrete predictions?

Working with EU

(Varian 1992) An individual has a vNM EU function over in which his utility numbers attached to outcomes defined by 'final wealth (w)' is given by $u(w) = \sqrt{w}$. He starts with \$4 and holds a lottery ticket that pays \$12 with probability $\frac{1}{2}$ and pays \$0 with probability $\frac{1}{2}$.

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$$EU = \frac{1}{2}u(4+12) + \frac{1}{2}u(4+0) = 3 \tag{5}$$

This is the same utility as if he had \$9 for sure. He'd sell the ticket for at least \$5.

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This is the same utility as if he had \$9 for sure. He'd sell the ticket for at least \$5. This example predicts our discussion of monetary lotteries and risk aversion. First we will look at some famous challenges to the model of choice that uses the EU form.

A thought experiment

Choose one of the following:

L_1: 100% chance of receiving \$1m.

L_1': 10% chance of receiving \$5m, 89% chance of receiving \$1m, 1% chance of receiving nothing.

A thought experiment

Choose one of the following:

L_2: 11% chance of receiving \$1m, 89% chance of receiving nothing.

L_2': 10% chance of receiving \$5m, 90% chance of receiving nothing.

The Allais paradox

If $L_1 \succ L_1'$ by an EU-maximizing DM:

$$u(1) > 0.1u(5) + 0.89u(1) + 0.01u(0) \quad (6)$$

$$\Rightarrow 0.11u(1) > 0.1u(5) + 0.01u(0) \quad (7)$$

$$\Rightarrow 0.11u(1) + 0.89u(0) > 0.1u(5) + 0.9u(0) \quad (8)$$

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$$\Rightarrow 0.11u(1) + 0.89u(0) > 0.1u(5) + 0.9u(0) \quad (8)$$

But if $L_2' \succ L_2$:

$$0.1u(5) + 0.9u(0) > 0.11u(1) + 0.89u(0) \quad (9)$$

An EU-maximizing DM cannot prefer L_1 to L_1' and L_2' to L_2?

The Allais paradox

Another example from Kahneman and Tversky (1979)

PROBLEM 1: Choose between

A:	2,500 with probability	.33,	B:	2,400 with certainty.
	2,400 with probability	.66,		
	0 with probability	.01;		
$N = 72$	[18]			[82]*

PROBLEM 2: Choose between

C:	2,500 with probability	.33,	D:	2,400 with probability	.34,
	0 with probability	.67;		0 with probability	.66.
$N = 72$	[83]*			[17]	

$$u(2{,}400) > .33u(2{,}500) + .66u(2{,}400) \text{ or } .34u(2{,}400) > .33u(2{,}500)$$

61% of subjects chose chose the modal response in both questions (i.e. true within-subject Allais violations)

Reactions to the Allais paradox

This is due to Allais (1953) and is an early example of an ‘empirical’ challenge to the validity of the EU model of choice under uncertainty. Motivator for **regret theory**.

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This is due to Allais (1953) and is an early example of an 'empirical' challenge to the validity of the EU model of choice under uncertainty. Motivator for **regret theory**. Some thoughts:

- Not all people in experiments display these preferences; one size does not seem to fit all.
- Did we have the right idea of what the 'final consequences' or 'outcomes' were? Semantics of journey vs. destination.
- Learning from one's 'mistakes'; e.g. Green (1987) Dutch books argument.
- Is the contradiction too stylized to generalize?
- Practical response 1: Relax independence axiom (procedural).
- Practical response 2: Define preferences over more than just 'final outcomes' (semantics).

Another thought experiment

An urn contains 300 balls. 100 are red, and 200 are either blue or green.
One ball will be drawn from the urn at random.
Choose one of the following:

L_1: Receive \$1,000 if the ball is red, else nothing.

L_1': Receive \$1,000 if the ball is blue, else nothing.

Another thought experiment

An urn contains 300 balls. 100 are red, and 200 are either blue or green. One ball will be drawn from the urn at random.
Choose one of the following:

L_2: Receive \$1,000 if the ball is not red, else nothing.

L_2': Receive \$1,000 if the ball is not blue, else nothing.

The Ellsberg paradox

Let $u(0) = 0$ for convenience. If $L_1 \succ L_1'$ by an EU-maximizing DM:

$$Pr(red)u(1000) > Pr(blue)u(1000) \quad (10)$$

$$\Rightarrow Pr(red) > Pr(blue) \quad (11)$$

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$$\Rightarrow Pr(red) > Pr(blue) \tag{11}$$

But if $L_2' \succ L_2$:

$$Pr(\neg red)u(1000) > Pr(\neg blue)u(1000) \tag{12}$$
$$\Rightarrow Pr(\neg red) > Pr(\neg blue) \tag{13}$$

Are the respondents thinking about these probabilities? What are they thinking about?

Reactions to the Ellsberg paradox

This is due to Ellsberg (1961). It is obviously similar in spirit, but where Allais pushes us to consider how the DM values objects, Ellsberg invites us to consider how the DM perceives probabilities.

- This is a motivation of the study of **ambiguity aversion**: "people prefer to act on events they feel well-informed about" (Ghirardato & Le Breton 2000)
- There is a connection with Knight's (1921) distinction between risk and uncertainty; it seems to matter whether we are considering concrete probabilities
- To incorporate ambiguity aversion, Schmeidler (1989) develops an EU representation with non-additive decision weights

Machina's paradox

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Is it crazy to prefer B to A?

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- Disappointment
- Preferences contingent on an unrealized outcome: failure of independence axiom?

Framing

Perhaps most problematic of all is due to Tversky and Kahneman (1986): An outbreak of a disease will cause 600 deaths. Two mutually exclusive responses are available.

Choice 1

A. 200 people will be saved
B. With probability $\frac{1}{3}$, 600 people will be saved; with probability $\frac{2}{3}$, 0 people will be saved.

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D. With probability $\frac{1}{3}$, 0 people will die; with probability $\frac{2}{3}$, 600 people will die.

72% chose A over B; from a different set of subjects 78% chose D over C. The choices are identical!

Framing

Again we see a situation where reference points matter, but this time they are exclusively primed

- The standard interpretation is that the DM is prompted to think in terms of lives lost vs. lives saved
- This is another indication that losses and gains might be processed in different ways

Money lotteries

- Back to the St. Petersburg paradox. The game pays an infinite amount of money! Why not pay very large amounts to play?
- Daniel Bernoulli (1738) argued in effect that postulating that people evaluate this game by thinking about expected money outcomes was not such a good *model* as one in which people evaluated the 'utility' of money outcomes. (see http://cerebro.xu.edu/math/Sources/NBernoulli/correspondence_petersb for a neat archive of preceding correspondence between Daniel, his brother Nicolas and others)
- Bernoulli argued for the plausibility of what we now call diminishing marginal utility of money.

Risk aversion

The EU theorem can be fit to outcomes that are defined by a continuous variable. Take lottery space $\mathcal{L}$ to be the set of all distribution functions over nonnegative amounts of money. A vNM utility function $U(\cdot)$ now looks like this:

$$U(F) = \int u(x)dF(x) \tag{14}$$

It's the mathematical expectation of the values of $u(x)$, which replaces the values $(u_1, ..., u_N)$ from before.

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- $u(\cdot)$ is the **Bernoulli utility function**
- Notice that the EU axioms *do not restrict it in any way*, so the onus is on the modeler (i.e. you) to specify interesting and relevant aspects of choice behavior
- For example: let's assume $u(\cdot)$ is increasing and continuous; the St. Petersburg paradox suggests also boundedness

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We can illustrate these all geometrically by plotting the Bernoulli function.

Risk aversion

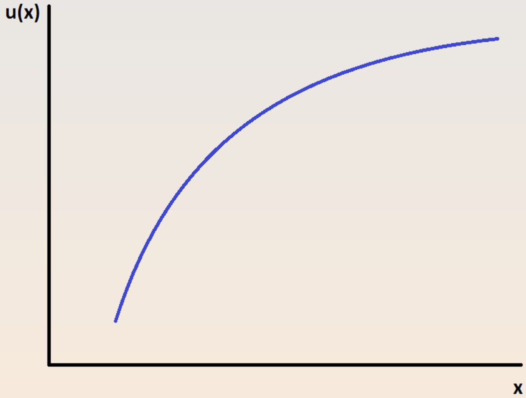

Figure: Bernoulli function for a risk averse DM

Risk aversion

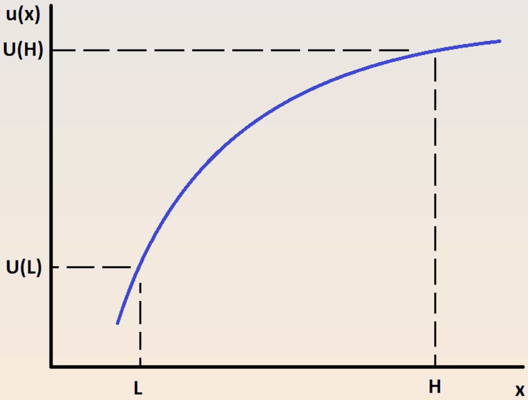

Figure: Two amounts, high and low, and the DM's utility from each

Risk aversion

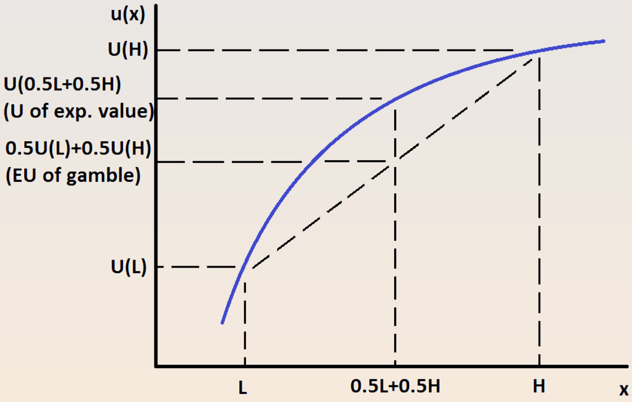

Figure: DM prefers to get the expected value of a gamble for sure rather than the gamble

Risk aversion

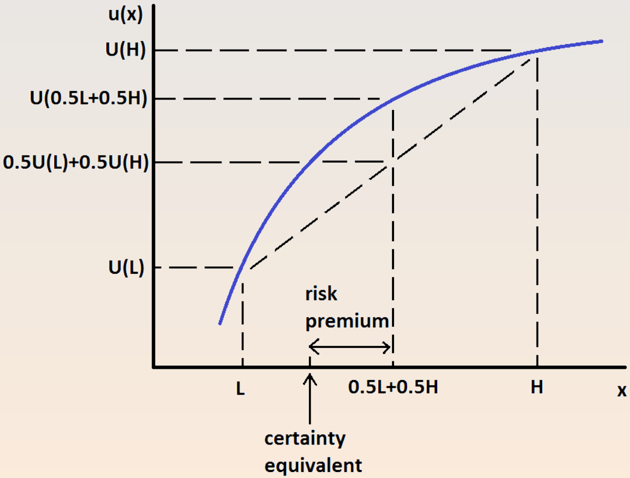

Figure: Amount of curvature captures DM's risk tolerance

Risk aversion

What about the *degree* of risk aversion?

Absolute risk aversion

The **Arrow-Pratt coefficient of absolute risk aversion** at x is $r_A(x) = -\frac{u''(x)}{u'(x)}$.

This is a measure of the curvature of $u(\cdot)$. If it's decreasing in x an individual will choose to take more risk at higher wealth levels.

Risk aversion

Absolute risk aversion evaluates attitudes toward gambles over absolute gains and losses. For gambles over *percentage* gains and losses:

Relative risk aversion

The **coefficient of relative risk aversion** at x is $r_R(x) = -\frac{xu''(x)}{u'(x)}$.

Nonincreasing relative risk aversion means that an individual becomes less risk averse with regard to gambles that are proportional to his wealth as his wealth increases.

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- Aumann & Serrano (2008): risk aversion captures how averse an individual is to risk, but not how risky the gamble is; like subjective time perception ("this movie was too long") without an objective measure of time ("3 hours")

Special cases of risk attitudes

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To see why:

$$u'(x) = \alpha e^{-\alpha x} \tag{16}$$

$$u''(x) = -\alpha^2 e^{-\alpha x} \tag{17}$$

$$r_A(x) = -\frac{u''(x)}{u'(x)} = \alpha \tag{18}$$

- The simple form for $r_A(x)$ makes this a convenient one to work with

Special cases of risk attitudes

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To see why:

$$u'(x) = x^{-\rho} \tag{20}$$

$$u''(x) = -\rho x^{-\rho-1} \tag{21}$$

$$r_A(x) = -\frac{xu''(x)}{u'(x)} = \rho \tag{22}$$

- For $\rho = 1$ we have an even more special case of CRRA: $u(x) = \ln x$

Comparing risk aversion

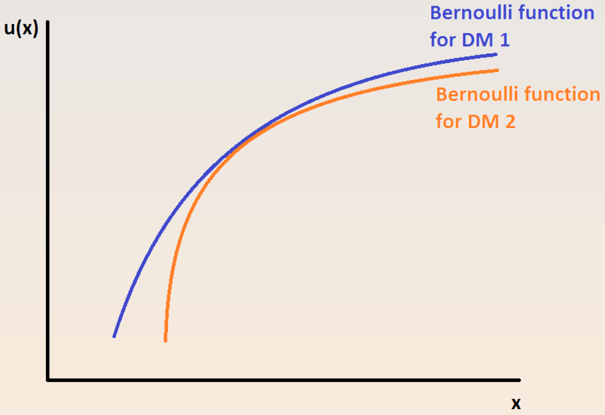

Figure: More curvature of Bernoulli function means more risk averse

Aumann and Serrano (2008)

Authors propose a way to measure the riskiness of a gamble

- A gamble here is a random variable with a positive expectation but some negative values
- For some gamble, find the CARA utility function such that the DM would be indifferent between taking the gamble or not
- The riskiness of a gamble is the reciprocal of the absolute risk aversion of that DM

Stochastic dominance

Take two random variables, x and y, with cumulative distribution functions F and G respectively

- We say that F **first order stochastically dominates** G if every vN-M EU maximizer with monotone preferences prefers F to G
  - Equivalently: F FOSD G if and only if $F(\eta) \leq G(\eta)$ for every η
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- Say that F and G have the same mean. We say that x **second order stochastically dominates** y if every risk-averse vN-M EU maximizer prefers F to G
  - Equivalently: G is a mean-preserving spread of F
  - Equivalently: $\int_a^\eta G(x)dx \geq \int_a^\eta F(x)dx$ for all $\eta \geq 0$

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- Say that F and G have the same mean. We say that x **second order stochastically dominates** y if every risk-averse vN-M EU maximizer prefers F to G
  - Equivalently: G is a mean-preserving spread of F
  - Equivalently: $\int_a^\eta G(x)dx \geq \int_a^\eta F(x)dx$ for all $\eta \geq 0$
- The problem with these concepts is that they aren't general enough: lots of things cannot be compared by these measures
- The same is true of heuristics like 'see if it has a higher mean' or 'see if its worse outcome is better than the other option's best outcome'
- Hence the need for something like Aumann-Serrano

Risk aversion and expected utility

Rabin (2000) and subsequently Rabin (2001) and Rabin & Thaler (2001) invoke risk aversion to challenge expected utility theory. Would you accept a gamble that gave a 50-50 chance of winning \$11 or losing \$10?

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- Curvature of the Bernoulli function required to explain a "risk-averse EU maximizer" rejecting above gamble implies he also rejects a 50-50 gamble between losing \$100 and winning an infinite amount.
- Relies crucially on "lifetime wealth" being the object of interest: again assumption on "consequences" is an inescapable auxiliary to any statement one makes about expected utility theory
- Watt (2002), Palacios-Huerta & Serrano (2006): and if the DM was truly considering lifetime wealth, best empirical estimates of level of risk aversion are incompatible with rejection of the small-stakes gambles
- Accepting large gambles is not evidence that a DM is not a risk-averse EU maximizer unless the evidence for rejecting small gambles is compelling

Demand for insurance

Consider a risk-averse expected utility maximizer who is deciding whether to buy insurance. She has initial wealth W, and with probability $\frac{1}{2}$ she will suffer a loss of L. Insurance is available that covers the loss when it occurs; this insurance costs C.

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Assume outcome is final wealth. What is her expected utility if she buys the insurance? If she doesn't buy? If $C = \frac{1}{2}L$, will she buy?

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$$EU(buy) = \frac{1}{2}u(W - L + L - C) + \frac{1}{2}u(W - C) \tag{23}$$

$$= u(W - C) \tag{24}$$

$$EU(don't) = \frac{1}{2}u(W - L) + \frac{1}{2}u(W) \tag{25}$$

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$$EU(don't) = \frac{1}{2}u(W - L) + \frac{1}{2}u(W) \quad (25)$$

Since the DM is risk averse, she will buy if $C = \frac{1}{2}$.

Demand for insurance

More general version of the same problem. She has initial wealth W, and there is some probability p that she will suffer a loss of L. Insurance is available that pays \$$q$ in the event of a loss, and the premium per dollar of coverage is π, so that q dollars of coverage costs \$$\pi q$. Assume that insurance companies are competitive and so make zero profit.

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How much coverage (q) will she buy?

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How much coverage (q) will she buy?

This is quite a complicated problem, so we will proceed in steps (in an exam you would be asked each step in turn).

Demand for insurance

Utility maximization problem (remember consumer chooses q):

$$\max_q EU = pu(W - L - \pi q + q) + (1-p)u(W - \pi q) \tag{26}$$

Derivative with respect to q, set equal to zero; optimal choice q^* solves:

$$pu'(W - L + q^*(1-\pi))(1-\pi) - (1-p)u'(W - \pi q^*)\pi = 0 \tag{27}$$

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$$\Rightarrow \frac{u'(W - L + q^*(1-\pi))}{u'(W - \pi q^*)} = \frac{(1-p)}{p}\frac{\pi}{1-\pi} \tag{28}$$

Demand for insurance

Firm's expected profit is:

$$(1-p)\pi q - p(\pi q - q) \tag{29}$$

Demand for insurance

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Zero profit assumption implies:

$$(1-p)\pi q - p(\pi q - q) = 0 \tag{30}$$

$$\pi - \pi p = p - \pi p \tag{31}$$

$$\pi = p \tag{32}$$

Zero profit for insurer implies **actuarially fair** premium: cost of policy is equal to expected value.

Demand for insurance

Putting that in the consumer's first order condition:

$$\frac{u'(W - L + q^*(1-\pi))}{u'(W - \pi q^*)} = \frac{(1-p)}{p} \frac{\pi}{1-\pi} \tag{33}$$

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Since a strictly risk-averse consumer has $u'' < 0$, this implies:

$$W - L + q^*(1-\pi) = W - \pi q^* \tag{35}$$

$$\Rightarrow q^* = L \tag{36}$$

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$$\Rightarrow q^* = L \tag{36}$$

The consumer insures against all losses.

Demand for insurance

So what?

- Note first that EU makes this little toy model tractable
- Firm implicitly risk-neutral, consumer risk-averse: *socially cheaper* for the firm to bear the whole risk
- But what if consumer's actions could affect the probability of loss?

Measuring risk aversion in the lab

Key considerations in getting risk aversion in the course of an experiment

- Might be concerned that hypothetical responses are different than real cash stakes
- Might be concerned that responses with small stakes wouldn't generalize upwards
- Might be concerned that the order in which you ask questions matters

Measuring risk aversion in the lab

Holt and Laury (2002) approach to getting at experimental subjects' risk aversion: run treatments with the following stakes but also 20x, 50x, and 90x these stakes, with both hypothetical and real treatments

Option A	Option B	Expected payoff difference
1/10 of $2.00, 9/10 of $1.60	1/10 of $3.85, 9/10 of $0.10	$1.17
2/10 of $2.00, 8/10 of $1.60	2/10 of $3.85, 8/10 of $0.10	$0.83
3/10 of $2.00, 7/10 of $1.60	3/10 of $3.85, 7/10 of $0.10	$0.50
4/10 of $2.00, 6/10 of $1.60	4/10 of $3.85, 6/10 of $0.10	$0.16
5/10 of $2.00, 5/10 of $1.60	5/10 of $3.85, 5/10 of $0.10	−$0.18
6/10 of $2.00, 4/10 of $1.60	6/10 of $3.85, 4/10 of $0.10	−$0.51
7/10 of $2.00, 3/10 of $1.60	7/10 of $3.85, 3/10 of $0.10	−$0.85
8/10 of $2.00, 2/10 of $1.60	8/10 of $3.85, 2/10 of $0.10	−$1.18
9/10 of $2.00, 1/10 of $1.60	9/10 of $3.85, 1/10 of $0.10	−$1.52
10/10 of $2.00, 0/10 of $1.60	10/10 of $3.85, 0/10 of $0.10	−$1.85

175 subjects (half undergrads, one third MBA students, 17% business school faculty) at 3 universities for lower stakes; 37 subjects for the higher stakes experiments at 1 university

Holt and Laury (2002)

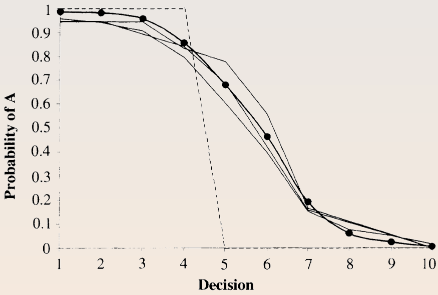


Figure 1. Proportion of Safe Choices in Each Decision: Data Averages and Predictions

Note: Data averages for low real payoffs [solid line with dots], 20x, 50x, and 90x hypothetical payoffs [thin lines], and risk-neutral prediction [dashed line].

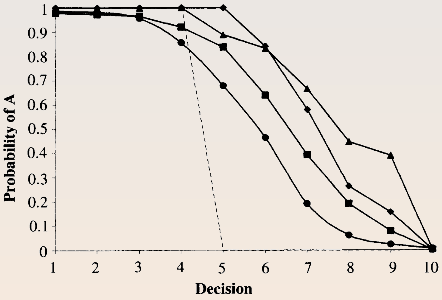


Figure 2. Proportion of Safe Choices in Each Decision: Data Averages and Predictions

Note: Data averages for low real payoffs [solid line with dots], 20x real [squares], 50x real [diamonds], 90x real payoffs [triangles], and risk-neutral prediction [dashed line].

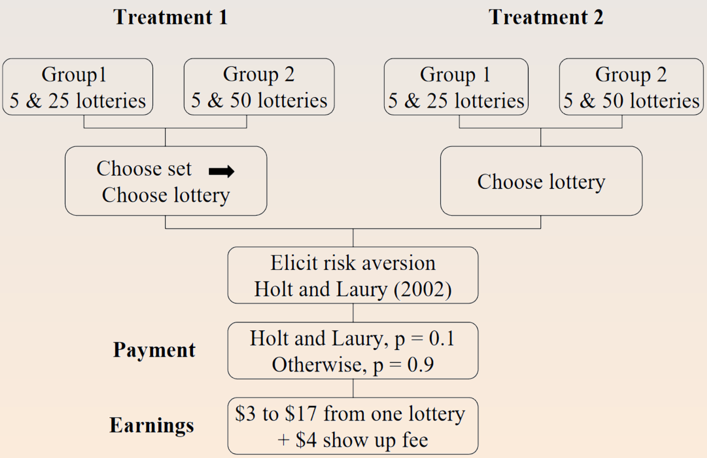
An example from Salgado (2006)

An example of a Holt-Laury choice table:

	Option 1		Option 2		
	\$12	**\$6**	**\$16**	**\$3**	**Expected Payoff**
Round	**Prob**	**Prob**	**Prob**	**Prob**	**Difference**
1	0.1	0.9	0.1	0.9	2.30
2	0.2	0.8	0.2	0.8	1.60
3	0.3	0.7	0.3	0.7	0.90
4	0.4	0.6	0.4	0.6	0.20
5	0.5	0.5	0.5	0.5	-0.50
6	0.6	0.4	0.6	0.4	-1.20
7	0.7	0.3	0.7	0.3	-1.90
8	0.8	0.2	0.8	0.2	-2.60
9	0.9	0.1	0.9	0.1	-3.30
10	1	0	1	0	-4.00

An example from Salgado (2006)

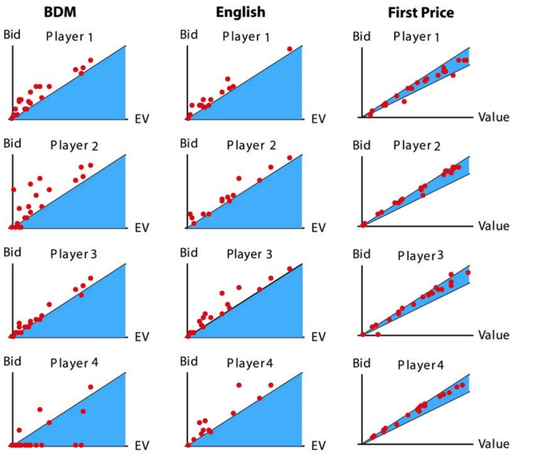
Position in the overall experimental design (note prob. of payout):



Is risk aversion a stable characteristic?

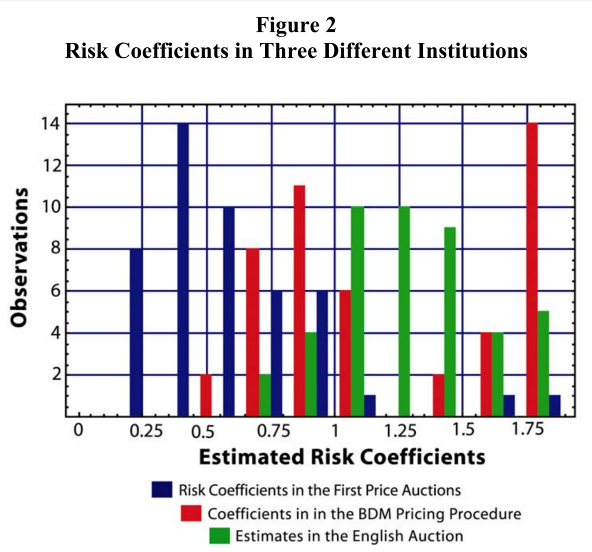
- Berg, Dickhaut, and McCabe (2005): measured risk preference may vary according to the institution
  - U Minnesota business school undergrads
  - Three institutions; value of prize randomly drawn for each subject
    1. BDM procedure to elicit selling price for a gamble
    2. English clock auction to elicit selling price for a gamble
    3. First-price auction to elicit bids for an object
- Hannah Schildberg-Hörisch (2018) summarizes work on how risk preferences change over a person's life cycle

Berg, Dickhaut, and McCabe (2005)



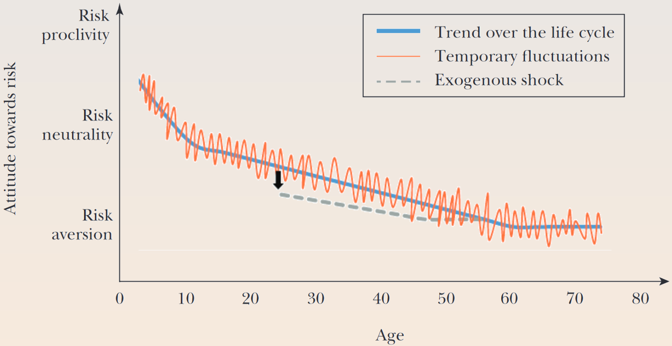
Each row in Figure 1 shows bidding behavior in the Becker, DeGroot, and Marschack pricing procedure, the English Auction and the First Price Auctions. Points (in red) show specific bids. For the BDM procedure and English Auction bids are graphed against Expected Value (EV). In the first price the bid of the winning bidder is graphed against value. In the BDM (and English) procedure a bid is associated with the related expected value of the bet to the subject. In the first price auction the bid of the winning bidder is graphed against the value for that subject. Shaded areas denote risk aversion.

Berg, Dickhaut, and McCabe (2005)



Schildberg-Hörisch (2018)

Illustration of the Framework for Studying the Stability of Risk Preferences



Note: This figure illustrates a framework for studying several possible reasons why an individual's risk aversion may change. The solid line shows continuous change in the mean-level of risk preferences, reflecting empirical evidence that individuals become more risk-averse over the life cycle. The dashed line represents a possible downward shift of the solid line for abrupt mean-level changes in individual risk preferences—as observed in the presence of exogenous shocks like economic crises. The jagged line represents temporary variation in risk preferences, in line with empirical evidence that temporary variations in emotions, self-control, or stress cause temporary variation in risk preferences around a baseline or average level.

Prospect Theory

Prospect Theory is the most famous example of a model that relaxes the restrictions on preferences (Kahneman & Tversky 1979). It begins from their interpretation of some observations from choice experiments:

- "Certainty effect": people overweigh certain outcomes relative to 'probable' outcomes
- "Reflection effect": people think in terms of gains and losses; risk aversion over gains mirrored by risk *loving* over losses - people overweigh certain *losses*
- "Isolation effect": simplify choice problem by disregarding 'common' (common-ish?) components

Use these 'violations' to reformulate the mechanics of a maximizing model.

Certainty effect

PROBLEM 3:

A:	(4,000,.80),	or	B:	(3,000).
$N = 95$	[20]			[80]*

PROBLEM 4:

C:	(4,000,.20),	or	D:	(3,000,.25).
$N = 95$	[65]*			[35]

Figure: Reducing prob. from 1 to 0.25 has a bigger effect than same proportionate reduction from 0.8 to 0.2

Certainty effect

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Problem 4:

C: (4,000,.20),		or	D: (3,000,.25).	
$N = 95$	[65]*			[35]

Figure: Reducing prob. from 1 to 0.25 has a bigger effect than same proportionate reduction from 0.8 to 0.2

- In Russian roulette, would you pay more to reduce the number of bullets from 4 to 3 as you would to reduce the number from 1 to 0?

Reflection effect

PREFERENCES BETWEEN POSITIVE AND NEGATIVE PROSPECTS

Positive prospects				Negative prospects			
Problem 3:	(4,000, .80)	<	(3,000).	Problem 3′:	(−4,000, .80)	>	(−3,000).
$N = 95$	[20]		[80]*	$N = 95$	[92]*		[8]
Problem 4:	(4,000, .20)	>	(3,000, .25).	Problem 4′:	(−4,000, .20)	<	(−3,000, .25).
$N = 95$	[65]*		[35]	$N = 95$	[42]		[58]
Problem 7:	(3,000, .90)	>	(6,000, .45).	Problem 7′:	(−3,000, .90)	<	(−6,000, .45).
$N = 66$	[86]*		[14]	$N = 66$	[8]		[92]*
Problem 8:	(3,000, .002)	<	(6,000, .001).	Problem 8′:	(−3,000, .002)	>	(−6,000, .001).
$N = 66$	[27]		[73]*	$N = 66$	[70]*		[30]

Figure: Subjects on average display risk aversion for positive prospects and risk loving for negative ones

Reflection effect

PROBLEM 11: In addition to whatever you own, you have been given 1,000. You are now asked to choose between

A: (1,000, .50), and B: (500).

$N = 70$ [16] [84]*

PROBLEM 12: In addition to whatever you own, you have been given 2,000. You are now asked to choose between

C: (−1,000, .50), and D: (−500).

$N = 68$ [69*] [31]

Figure: ...even when the final cash position is the same (framing, thinking in terms of gains and losses)

Reflection effect

PROBLEM 13:

(6,000, .25), or (4,000, .25; 2,000, .25).

$N = 68$ [18] [82]*

PROBLEM 13′:

(−6,000, .25), or (−4,000, .25; −2,000, .25).

$N = 64$ [70]* [30]

Applying equation 1 to the modal preference in these problems yields

$$\pi(.25)v(6{,}000) < \pi(.25)[v(4{,}000) + v(2{,}000)] \quad \text{and}$$

$$\pi(.25)v(-6{,}000) > \pi(.25)[v(-4{,}000) + v(-2{,}000)].$$

Hence, $v(6{,}000) < v(4{,}000) + v(2{,}000)$ and $v(-6{,}000) > v(-4{,}000) + v(-2{,}000)$.

Isolation effect

PROBLEM 7:

A: (6,000, .45), B: (3,000, .90).

$N = 66$ [14] [86]*

PROBLEM 8:

C: (6,000, .001), D: (3,000, .002).

$N = 66$ [73]* [27]

Figure: Small probabilities not well understood, overweighed... 0.001 and 0.002 'look similar' so the DM focuses on the prize amount

Small probabilities, small insurance

'Overweighting' of small probabilities (distinct from overestimation of prob. of rare events)

PROBLEM 14:

	(5,000, .001),	or	(5).
$N = 72$	[72]*		[28]

PROBLEM 14′:

	(−5,000, .001),	or	(−5).
$N = 72$	[17]		[83]*

Figure: 14: prefer lottery ticket over expected value of the ticket; 14': prefer a small loss over a small chance of large loss

Prospect Theory

Idea is that people use an "editing" phase to understand and simplify the choice in their mind; "evaluation" phase to decide on something

- Value of edited prospect expressed in terms of π and v.
- π: associated with each probability p a decision weight $\pi(p)$. Impact of p on value of prospect.
- v: assigns to each outcome x a number $v(x)$, subjective value of the outcome. Outcomes are deviations from a reference point: v measures gains and losses.

Think of prospects $(x, p; y, q)$: at most two non-zero outcomes. x with probability p, y with probability q, nothing with probability $1 - p - q$, $p + q \leq 1$.

Prospect Theory

If $(x, p; y, q)$ is a *regular* prospect $(p + q < 1,\ x \geq 0 \geq y$ or $x \leq 0 \leq y)$, then

$$V(x, p; y, q) = \pi(p)v(x) + \pi(q)v(y), \qquad (37)$$

where $v(0) = 0$, $\pi(1) = 1$, $\pi(0) = 0$.

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If $p + q = 1$ and either $x > y > 0$ or $x < y < 0$, then

$$V(x, p; y, q) = v(y) + \pi(p)[v(x) - v(y)]. \tag{38}$$

Certain component plus value-difference.

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Certain component plus value-difference.

What do the functions π and v look like?

The value function

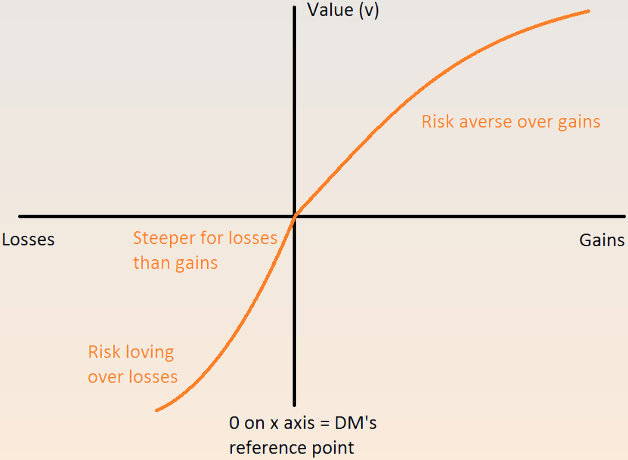


Figure: General shape of Prospect Theory value function

The decision weight function

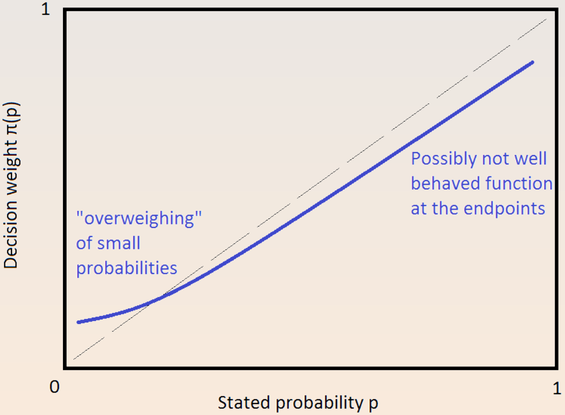


Figure: General shape of Prospect Theory decision weights

Gonzalez & Wu (1999)

You have two lotteries to win $250. One offers a 5% chance to win the prize and the other offers a 30% chance to win the prize.

A: You can improve the chances of winning the first lottery from 5 to 10%.

B: You can improve the chances of winning the second lottery from 30 to 35%.

Which of these two improvements, or increases, seems like a more significant change? (circle one)

You have two lotteries to win $250. One offers a 65% chance to win the prize and the other offers a 90% chance to win the prize.

C: You can improve the chances of winning the first lottery from 65 to 70%.

D: You can improve the chances of winning the second lottery from 90 to 95%.

Which of these two improvements, or increases, seems like a more significant change? (circle one)

75% picked A; 37% picked C (ordering of the two questions counterbalanced)

Estimating the decision weight function

Gonzalez & Wu (1996, 1999): calibration of decision weight function with experimental evidence

- How to handle non-linear probability weighting?
- Innovation is to avoid having to make assumptions about the exact functional form of the value and decision weight functions

Estimating the decision weight function

Gonzalez & Wu (1996, 1999): calibration of decision weight function with experimental evidence

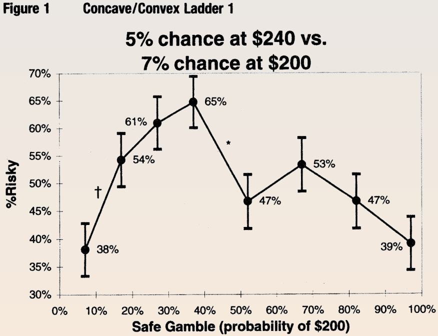
- How to handle non-linear probability weighting?
- Innovation is to avoid having to make assumptions about the exact functional form of the value and decision weight functions
- Boil the issue down to common-consequence conditions on concavity vs. convexity of the decision weight function
- Test concavity vs. convexity with ‘ladders’ that add common consequences to subsequent choices
- Each subject saw four of the eight ‘rungs’; order of questions randomized; order of options counter-balanced
- Between and within subject analyses possible

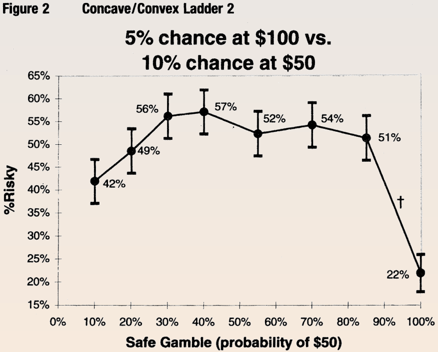
Gonzalez & Wu (1996)

Table 1 Concavity/Convexity Ladders

	Rung 1	Rung 2	Rung 3	Rung 4	Rung 5	Rung 6	Rung 7	Rung 8
				Ladder 1				
R	0.05, $240	0.05, $240	0.05, $240	0.05, $240	0.05, $240	0.05, $240	0.05, $240	0.05, $240
		0.10, $200	0.20, $200	0.30, $200	0.45, $200	0.60, $200	0.75, $200	0.90, $200
S	0.07, $200	0.17, $200	0.27, $200	0.37, $200	0.52, $200	0.67, $200	0.82, $200	0.97, $200
				Ladder 2				
R	0.05, $100	0.05, $100	0.05, $100	0.05, $100	0.05, $100	0.05, $100	0.05, $100	0.05, $100
		0.10, $50	0.20, $50	0.30, $50	0.45, $50	0.60, $50	0.75, $50	0.90, $50
S	0.10, $50	0.20, $50	0.30, $50	0.40, $50	0.55, $50	0.70, $50	0.85, $50	$50
				Ladder 3				
R	0.01, $300	0.01, $300	0.01, $300	0.01, $300	0.01, $300	0.01, $300	0.01, $300	0.01, $300
		0.10, $150	0.20, $150	0.30, $150	0.45, $150	0.60, $150	0.80, $150	0.98, $150
S	0.02, $150	0.12, $150	0.22, $150	0.32, $150	0.47, $150	0.62, $150	0.82, $150	$150
				Ladder 4				
R	0.03, $320	0.03, $320	0.03, $320	0.03, $320	0.03, $320	0.03, $320	0.03, $320	0.03, $320
		0.10, $200	0.20, $200	0.30, $200	0.45, $200	0.65, $200	0.85, $200	0.95, $200
S	0.05, $200	0.15, $200	0.25, $200	0.35, $200	0.50, $200	0.70, $200	0.90, $200	$200
				Ladder 5				
R	0.01, $500	0.01, $500	0.01, $500	0.01, $500	0.01, $500	0.01, $500	0.01, $500	0.01, $500
		0.10, $100	0.20, $100	0.30, $100	0.45, $100	0.65, $100	0.80, $100	0.95, $100
S	0.05, $100	0.15, $100	0.25, $100	0.35, $100	0.50, $100	0.70, $100	0.85, $100	$100

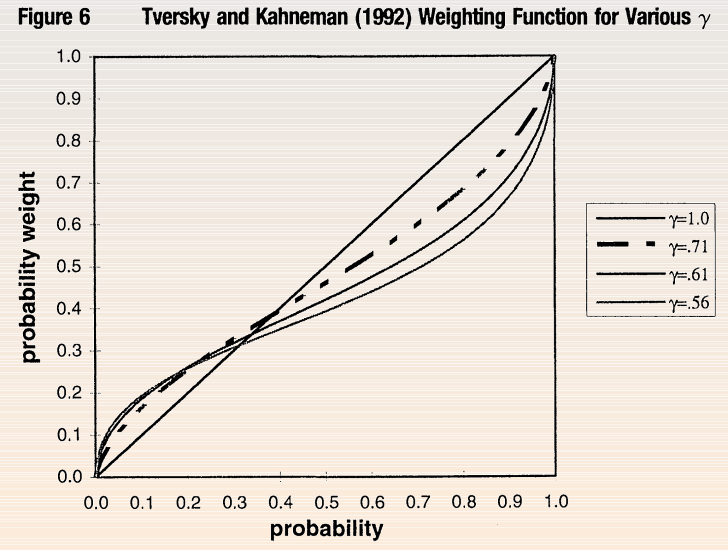
Gonzalez & Wu (1996)





Inverted U shape consistent with concave-then-convex decision weight function

Gonzalez & Wu (1996)



Gonzalez & Wu (1996)

- Decision weight function concave up to around $p = 0.4$ and convex after
- A linear functional form with discontinuities at the endpoints performs worse than a strictly nonlinear form
- The fit to Tversky & Kahneman (1992) and Prelec (1995) single-parameter functional forms yields a result that substantially improves on EUT in explaining choices

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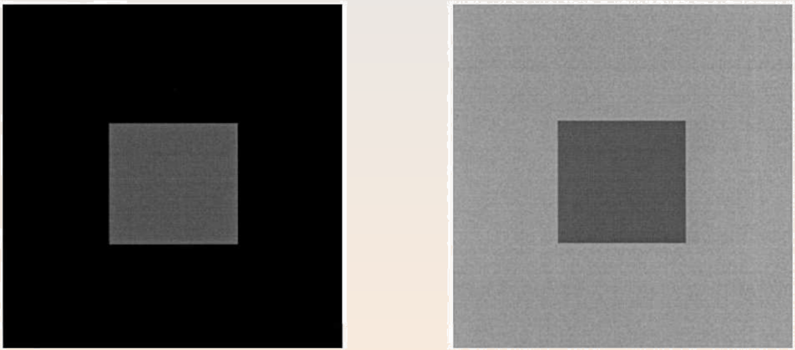
- The fit to Tversky & Kahneman (1992) and Prelec (1995) single-parameter functional forms yields a result that substantially improves on EUT in explaining choices
- Gonzalez & Wu (1999):
  - Two-parameter function: one controlling curvature and one controlling height of the probability weighting function
  - 'Discriminability': diminishing sensitivity to changes in probabilities away from the endpoints (curvature of the S)
    - Experts are more linear (Thaler & Ziembra 1988 horse race gamblers; Fox, Rogers, & Tversky 1996 options traders)
  - 'Attractiveness': for a given probability different DMs might find the gamble more or less attractive, i.e. over- or under-weight differently (height of the S)
  - Estimate this for different subjects in similar fashion to other paper

Reference points as human experience



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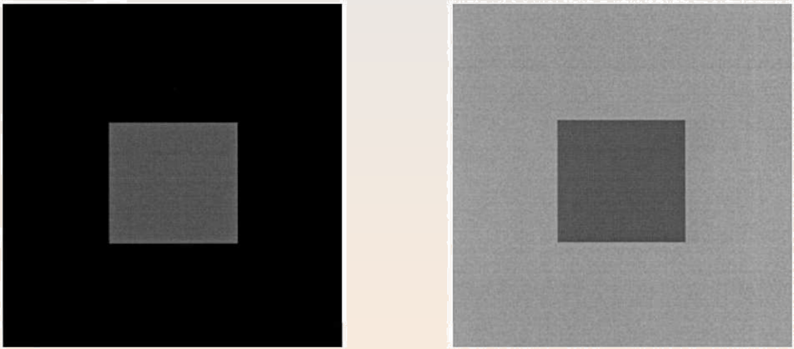

The two interior squares are the same color

Loss aversion

The ‘reflection effect’ is closely related to the general concept of **loss aversion**

Adam Smith, 1759

“Pain... is, in almost all cases, a more pungent sensation than the opposite and correspondent pleasure. The one almost always depresses us much more below the ordinary, or what may be called the natural state of our happiness, than the other ever raises us above it.”

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- Loss aversion: utility loss from losing something is more than the utility gain from getting it
- Unifying idea that can explain reflection effect, endowment effect, status quo bias (from our earlier notes)

A model of loss aversion

- Two things that the DM cares about: bundles have amounts (x_1, x_2)
- DM has a reference point for each of them: (r_1, r_2)

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- Utility will depend on amounts relative to reference points:

$$u_i(x_i - r_i) \text{ if } x_i > r_i \tag{39}$$
$$-\lambda u_i(r_i - x_i) \text{ if } x_i < r_i \tag{40}$$

- So $u_1(1) - \lambda u_2(1)$ is the utility of giving up one of good 2 to get one more of good 1

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- So $u_1(1) - \lambda u_2(1)$ is the utility of giving up one of good 2 to get one more of good 1
- Consider two possibilities:
  1. WTP for good 2: how much of good 1 would DM be willing to give up for one unit of good 2?
  2. WTA for good 2: how much of good 1 would DM be willing to accept in exchange for one unit of good 2?

A model of loss aversion

- Assume utility is linear in good 2
- DM is happy to pay z to receive a unit of good 1 if

$$u_2(1) - \lambda z > 0 \tag{41}$$

$$z < \frac{u_2(1)}{\lambda} \tag{42}$$

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$$u_2(1) - \lambda z > 0 \quad (41)$$

$$z < \frac{u_2(1)}{\lambda} \quad (42)$$

- DM is happy to accept y to give up a unit of good 1 if

$$-\lambda u_2(1) + y > 0 \quad (43)$$

$$y > \lambda u_2(1) \quad (44)$$

- Ratio of WTP to WTA is $\frac{z}{y} = \frac{1}{\lambda^2}$: WTP¡WTA when $\lambda > 1$

Loss aversion for cash gambles

- Assume utility is linear in a single good, cash
- Certainty equivalent of 50-50 gamble to gain $2z$ or gain 0

$$u(CE) = \frac{1}{2}u(2z) + \frac{1}{2}u(0) \tag{45}$$

$$CE = z \tag{46}$$

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$$CE = z \tag{46}$$

- Certainty equivalent of 50-50 gamble to gain z or lose z

$$u(CE) = \frac{1}{2}u(z) + \frac{1}{2}(-\lambda)u(z) \tag{47}$$
$$CE < 0 \text{ if } \lambda > 1 \tag{48}$$

- This λ method models the 'warp' around 0

Measuring loss aversion

- Dean and Ortoleva (2014): take same group of subjects and measure λ parameter in lottery problems and the WTP/WTA ratio of the endowment effect
- Measuring λ (uses CRRA parameterization; Abdellaoui et al. 2008)
  - Questions on risky bets used to estimate CRRA utility function for gains
  - Certainty equivalence of mirror lotteries in loss domain used to estimate CRRA utility function for losses
  - Elicit values of gain to make subject indifferent between \$0 and a lottery that pays the gain or a loss of \$6, \$8, or \$10 with 50-50 prob.
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- Measuring WTP/WTA gap:
  - WTP/WTA gap for buying vs. selling a lottery ticket that paid \$10 with 50% chance and \$0 with 50% chance
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  - Endow subjects with additional \$10 and elicit how much of the additional \$10 they would pay to buy the 50-50 lottery between \$10 and \$0 (WTP)
- Correlation of 0.63 between the two measures within subjects

Some examples of phenomena consistent with loss aversion

- Equity premium puzzle (Benartzi and Thaler 1995)
  - Return on stocks much higher than on bonds to an extent hard to explain with just risk aversion
  - Loss aversion plus 'narrow bracketing' (evaluating portfolio items separately rather than as a whole) can explain
  - Estimated λ from calibrated model of 2.25, similar to experiments

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- Disposition effect (Odean 1998)
  - Losing stocks held for median of 124 days vs. 104 days for winning stocks
  - But winners return the next year averages 11.6% vs. 5% for losers
  - Buying price becomes a reference point

Information aversion

An implication of loss aversion is that DM may try to avoid intermediate information

- Take $u(x) = x$ and $\lambda = 2.5$
- Gamble: two sequential 50-50 chances of $+200$ or -100 (Samuelson 1963)

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- Take $u(x) = x$ and $\lambda = 2.5$
- Gamble: two sequential 50-50 chances of +200 or -100 (Samuelson 1963)
- Utility depends on whether you learn the intermediate result or just the aggregate

$$\frac{1}{4}(200+200)+\frac{1}{2}(200-\lambda 100)+\frac{1}{4}(-\lambda 100-\lambda 100) = -200 \quad (49)$$

$$\frac{1}{4}(400)+\frac{1}{2}(100)+\frac{1}{4}(-\lambda 200) = 25 \quad (50)$$

- Check a risky asset less when times are turbulent (Andries and Haddad 2015)
- Observed in experimental setting in Gneezy and Potter (1997)

Gneezy & Potter (1997)

Average Percentage of Endowment Bet (Part 1)

	Treatment H[a]	Treatment L[a]	Mann-Whitney z[b]
Rounds 1–3	52.0 (30.2)	66.7 (29.5)	−2.08 [0.018]
Rounds 4–6	44.8 (30.0)	63.7 (30.3)	−2.78 [0.003]
Rounds 7–9	54.7 (28.9)	71.9 (29.4)	−2.51 [0.006]
Rounds 1–9	50.5 (26.7)	67.4 (27.3)	−2.86 [0.002]

a. # obs. = 41 (42) for treatment H (L). Standard deviations are in parentheses.
b. One-tailed significance levels (p-values) are in brackets.

Average Amount Bet, Average Percentage Bet, and Average Total Earnings

	Treatment H[a]	Treatment L[a]	Mann-Whitney z[b]
Amount bet (Y)	707.3 (614.5)	887.1 (662.1)	−2.14 [0.016]
Percentage bet (F)	39.0 (30.0)	48.9 (32.1)	−1.62 [0.053]
Total earnings (parts 1 and 2)	1822 (1015)	2134 (745)	−1.78 [0.038]

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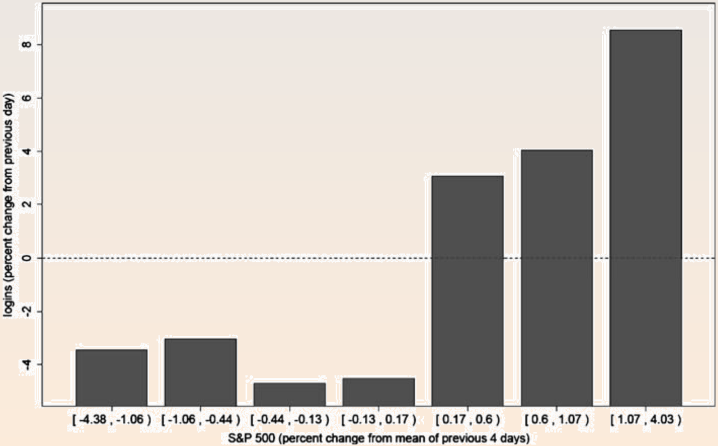
Karlsson, Loewenstein, and Seppi (2009)

'Ostrich effect': investors log in to their accounts less when stocks are down



Paper vs. realized losses

Imas (2016) dives a bit more into this by looking at risk-taking after paper losses versus realized losses

- 128 undergraduates; \$5 show up fee
- Endowed with \$8 in an envelope
- 4 rounds of investment decisions: how much of \$2 to invest in a lottery (25 cent increments); prob. $\frac{1}{6}$ of winning 7 times that amount or else lose it all
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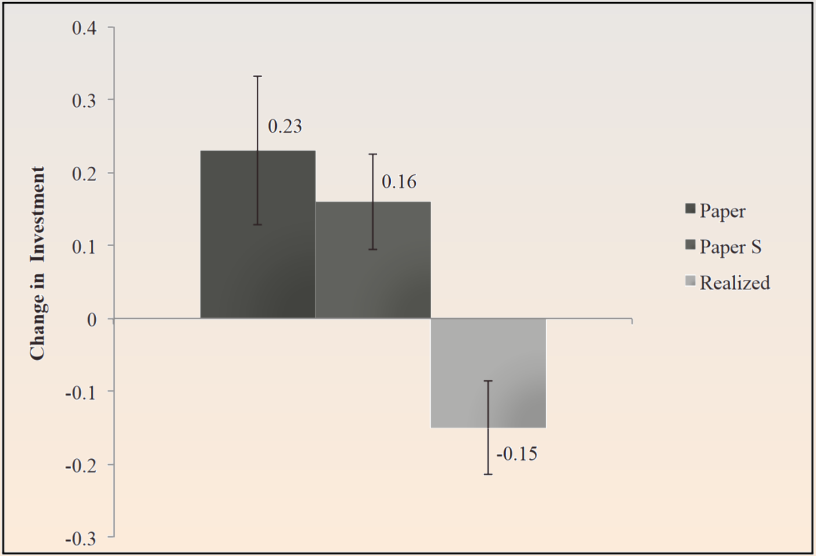
- Implemented by public dice roll; note positive expected payoff
- Treatments:
  1. Realized: money changed hands after round 3 before last round
  2. Paper: continued on to round 4 as normal (time between rounds equalized from realized treatment)
  3. Paper Social: informed verbally about cash position after round 3

Investment change after 3 losses



Paper vs. realized losses

- Next: some robustness checks
- Another lab experiment with Realized and Paper S but also a couple of new treatments
- 'Transfer' treatment: participants were not given envelope of cash at the start; after round 3 presented with their earnings up to that point
- 'Interrupt' treatment: similar to Paper S but had a 5 minute filler task between rounds 3 and 4

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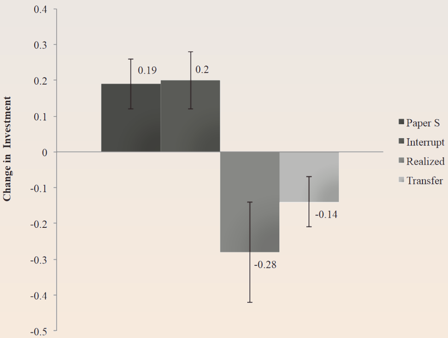
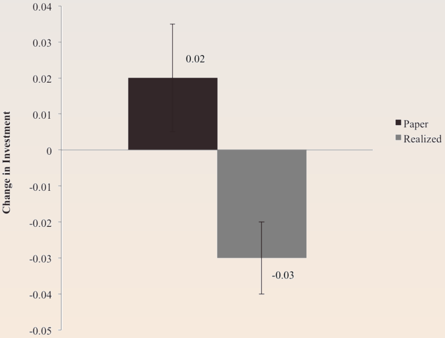
- An Amazon Mechanical Turk experiment with just Paper vs Realized

Investment change after 3 losses



(a) *Lab Experiment*



(b) *Mechanical Turk Experiment*

Reference dependence and effort

One possible way to identify reference dependence in data is via its effect on effort

- Define effort as e with cost $c(e)$, reference point r, relative weight on reference point η
- Above and below reference point individual maximizes:

$$\max_e e + \eta(e - r) - c(e) \text{ for } e \geq r \quad (51)$$

$$\max_e e + \eta\lambda(e - r) - c(e) \text{ for } e < r \quad (52)$$

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- Optimal choice will balance marginal benefit and marginal cost of effort:

$$1 + \eta = c'(e^*) \text{ for } e \geq r \tag{53}$$

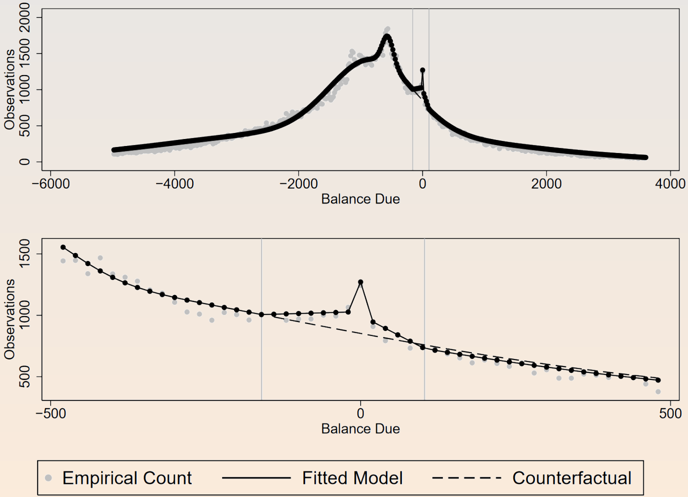
$$1 + \eta\lambda = c'(e^*) \text{ for } e < r \tag{54}$$

Reference dependence and effort

Effort has a higher marginal utility below the reference point than above

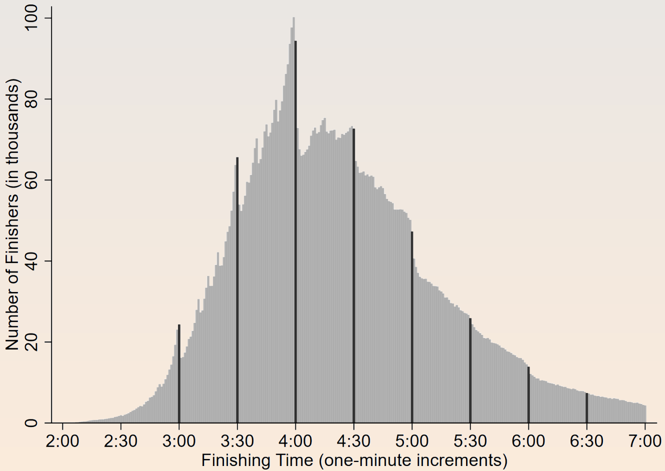
- Optimal 'easing off' above the reference point relative to 'pushing harder' below
- In data would show up as:
  1. Bunching at the reference point $e^* = r$
  2. Missing mass of the probability distribution below the reference point compared to above

Evidence of loss aversion in tax returns (Rees-Jones 2018)



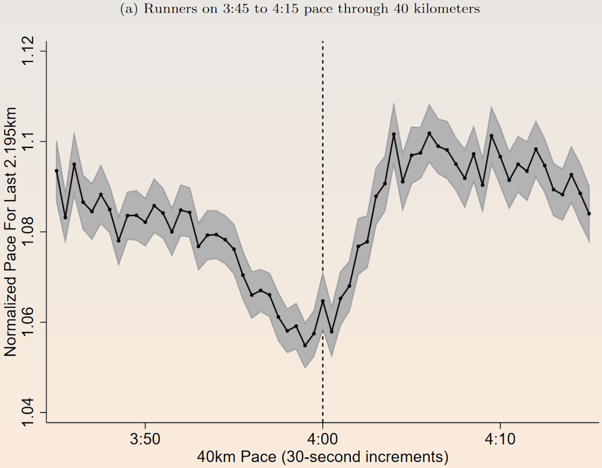
Spike at zero in U.S. tax return data consistent with loss aversion

Reference points for marathon runners (Allen et al. 2017)



Bunching at ‘round number’ target reference points

Reference points for marathon runners (Allen et al. 2017)



Runners near a reference point speed up or slow down near the end

Subjective Expected Utility

SEU is distinct from EU in situations of uncertainty rather than risk: probabilities are not known

- States of the world: an exclusive and exhaustive list of all the outcomes that could happen
- Acts: actions defined by what outcome they yield in each state

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SEU is a simple extension of EU:

1. Associate each state of the world with a probability
2. Associate each prize with a utility
3. Compute the Expected Utility of each act using the probabilities and utilities from steps 1 and 2
4. Choose the act with the highest Expected Utility

Subjective Expected Utility definition

Subjective Expected Utility

Let X be the set of prizes, Ω be a finite set of states, and F be the resulting set of acts $f : \Omega \to X$.

The preference relation $\succsim$ on F has a Subjective Expected Utility representation if there exists utility function $u : X \to \mathcal{R}$ and probability function $\pi : \Omega \to [0,1]$ such that $\sum_{\omega \in \Omega} \pi(\omega) = 1$ and $f \succsim g$ if and only if

$$\sum_{\omega \in \Omega} \pi(\omega) u(f(\omega)) \geq \sum_{\omega \in \Omega} \pi(\omega) u(g(\omega)) \tag{55}$$

Rank dependent utility

RDU gives a generalization of EU that allows the weight on a prize to depend on how good the prize is

- In RDU model the probability weighting used by the DM depends on two things:
  1. The probability with which a prize arrives
  2. The rank of the prize in the lottery relative to other prizes

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- In RDU model the probability weighting used by the DM depends on two things:
  1. The probability with which a prize arrives
  2. The rank of the prize in the lottery relative to other prizes
- The weight attached to the top prize is whatever the decision weight on its probability is
- The weight on *n*th best is the weight on probability of
  - getting something at least as good as it,
  - minus getting something better than it

Example (following Mark Dean's notes)

Lottery L with three prizes: 10 with probability 0.1, 5 with probability 0.7, 0 with probability 0.2

$$\begin{aligned} RDU(L) = & \psi(p_1)u(x_1) && (56)\\ & + (\psi(p_1+p_2) - \psi(p_1))u(x_2) && (57)\\ & + (\psi(p_1+p_2+p_3) - \psi(p_1+p_2))u(x_3) && (58)\\ & && (59) \end{aligned}$$

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& && (59)
\end{aligned}
$$

For this example:

$$
\begin{aligned}
RDU(L) = & \psi(0.1)u(10) && (60) \\
& + (\psi(0.8) - \psi(0.1))u(5) && (61) \\
& + (\psi(1) - \psi(0.8))u(0) && (62)
\end{aligned}
$$

If $\psi = 1$ we have EUT

Allais & RDU (following Mark Dean's notes)

Revisiting the Allais paradox:

1. What amount $x > 1,000,000$ would make DM indifferent between:

   L_1: 100% chance of receiving \$1m
   L_1': 10% chance of receiving x, 89% chance of receiving \$1m, 1% chance of receiving nothing

2. Choose one of the following:

   L_2: 11% chance of receiving \$1m, 89% chance of receiving nothing.
   L_2': 10% chance of receiving z, 90% chance of receiving nothing.

EUT requires $x = z$ while Allais manifests as $x > z$; let's assume $u(x) = x$ for convenience and check what RDU has to say

Allais & RDU

RDU of L_1:

$$\psi(1)1,000,000 = 1,000,000 \tag{63}$$

Allais & RDU

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RDU of L_1':

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RDU of L_1':

$$\psi(0.1)x + (\psi(0.99) - \psi(0.1))1,000,000 + (\psi(1) - \psi(0.99))0 \tag{64}$$

For DM to be indifferent between these:

$$1m > \psi(0.1)x + (\psi(0.99) - \psi(0.1))1m + (\psi(1) - \psi(0.99))0 \tag{65}$$

$$\Rightarrow 1m > \psi(0.1)x + (\psi(0.99) - \psi(0.1))1m \tag{66}$$

$$\Rightarrow x = \frac{1 - (\psi(0.99) - \psi(0.1))}{\psi(0.1)} 1m \tag{67}$$

Allais & RDU

RDU of L_2:

$$\psi(0.11)1,000,000 + (1 - \psi(0.11))0 \quad (68)$$

Allais & RDU

RDU of L_2:

$$\psi(0.11)1,000,000 + (1 - \psi(0.11))0 \tag{68}$$

RDU of L_2':

$$\psi(0.1)z + (1 - \psi(0.1))0 \tag{69}$$

Allais & RDU

RDU of L_2:

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RDU of L_2':

$$\psi(0.1)z + (1 - \psi(0.1))0 \tag{69}$$

For DM to be indifferent between these:

$$\psi(0.11)1,000,000 + (1 - \psi(0.11))0 = \psi(0.1)z + (1 - \psi(0.1))0 \tag{70}$$

$$\Rightarrow \psi(0.11)1,000,000 = \psi(0.1)z \tag{71}$$

$$\Rightarrow z = \frac{\psi(0.11)}{\psi(0.1)}1m \tag{72}$$

Allais & RDU

So Allais paradox manifests if:

$$x > z \tag{73}$$

$$\frac{1 - (\psi(0.99) - \psi(0.1))}{\psi(0.1)} > \frac{\psi(0.11)}{\psi(0.1)} \tag{74}$$

$$\psi(1) - \psi(0.99) > \psi(0.11) - \psi(0.1) \tag{75}$$

That is: the increase in decision weight from 99% to 100% is bigger than the weight of going from 10% to 11%

Allais & RDU

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That is: the increase in decision weight from 99% to 100% is bigger than the weight of going from 10% to 11%

- As an example: $\psi(p) = p^2$
- $1^2 - 0.99^2 \approx 0.0199 > 0.0021 \approx 1^2 - 0.11$

Allais & RDU

So Allais paradox manifests if:

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- $\psi(p) = p^1$: $1 - 0.99 = 0.01 > 0.01 = 0.11 - 0.1$
- $\psi(p) = p^{0.5}$: $1^{0.5} - 0.99^{0.5} \approx 0.005 < 0.015 \approx 0.11^{0.5} - 0.1^{0.5}$

Ellsberg evidence

Halevy (2007) conducts a series of experiments to explore the link between attitudes to ambiguity and to compound objective lotteries

- Four urns, each with 10 red or black balls
    1. 5 red and 5 black (risk)
    2. Unknown composition (ambiguity)
    3. Number of red balls uniformly distributed between 0 and 10
    4. Either 10 red or 10 black balls with equal probability

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- Subjects asked to bet on a color being drawn from each urn; win $2 if correct, lose nothing if wrong
- Then subjects had the option to sell each of the bets
  - Asked to state four minimum prices to sell, between $0 and $2
  - For each urn the computer chose a random number between $0 and $2
  - If random number was higher than the min. price, subject got the price for the sale; if not, their earnings depended on the bet

Maxmin Expected Utility

Gilboa and Schmeidler (1989) suggest a model that can explain ambiguity aversion

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- Premise is pessimistic: what's the worst case? What if the world is rigged against you?
- Think of the range of possible probability distributions that could apply
- Evaluate your chosen act using the worst of the distributions
- **Maxmin Expected Utility**: maximize the minimum utility you could get across different probability distributions

Maxmin Expected Utility: example

A modified Halvey example:

- Choice 1: risky bag
  - Bag has 20 red and 20 black chips
  - DM makes a $10 bet on a color of their choice
  - Elicit amount $x such that DM is indifferent between keeping the bet and receiving $x

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- The Ellsberg manifestation here is that x is bigger than y: willing to take a smaller sure thing in exchange for the ambiguous bag
- The DM having subjective probability assessments here is not enough since one of the two cases (red or black) must have prob. of at least 0.5 in the ambiguous case

Maxmin Expected Utility: example

Let $u(x) = x$ and say that the DM holds a belief that the probability of a red ball in the ambiguous bag is between 0.25 and 0.75

- EU of bet on the risky bag is $\frac{1}{2}u(10) + \frac{1}{2}u(0) = 5$; MEU is the same since there is only one prob. distribution in play here
- MEU of bet on red from the ambiguous bag is given by

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- Max utility from bet on ambiguous bag is lower than the bet on the risky bag

Maxmin Expected Utility definition

Maxmin Expected Utility

Let X be the set of prizes, Ω be a finite set of states, and F be the resulting set of acts $f : \Omega \to X$.

The preference relation $\succsim$ on F has a Maxmin Expected Utility representation if there exists utility function $u : X \to \mathcal{R}$ and a convex set of probability functions Π such that $f \succsim g$ if and only if

$$\min_{\pi \in \Pi} \sum_{\omega \in \Omega} \pi(\omega) u(f(\omega)) \geq \min_{\pi \in \Pi} \sum_{\omega \in \Omega} \pi(\omega) u(g(\omega)) \tag{78}$$

MEU and 'no trade'

An application to finance: no trade price regions (Dow, James, and Werlang 1992)

- Asset with price p pays \$10 if company is successful or \$0 if not
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Say that the DM thinks there's a range of possible probabilities of success: from a low of $\pi_*(s)$ to a strictly higher high of $\pi^*(s)$

MEU and 'no trade'

Option 1: buy the asset

- Use the probability $\pi_*(s)$, the worst-case probability of success

$$MEU(buy) = \pi_*(s)(10 - p) + (1 - \pi_*(s))(-p) > 0 \quad (79)$$

$$\text{if } p < 10\pi_*(s) \quad (80)$$

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In the region $10\pi_*(s) < p < 10\pi^*(s)$ the DM will neither buy nor sell: this is the no trade region for the price of this asset for this DM